The 1st Workshop on DC

Developments in Combinatorics (DC)

October 26-27, 2021, Beijing

Zoom ID: 890 901 6918 **Password: 202121**

13:00-13:10	**Opening Ceremony** **Organizers:** **Hong Liu** (University of Warwick) **Guanghui Wang** (Shandong University)	
Oct. 26 **Time**	**Lecturer**	**Title**
13:10-13:45	Felix Clemen (University of Illinois Urbana-Champaign)	Maximum number of almost similar triangles in the plane
13:45-14:20	Shumin Sun (Shandong University)	F-factors in quasi-random hypergraphs
Open Problem (1 hour)		
15:20-15:55	Tuan Tran (Institute for Basic Science)	Exponential decay of intersection volume with applications on list-decodability and Gilbert--Varshamov type bound
15:55-16:30	Stefan Glock (ETH Zürich)	New results for MaxCut in H-free graphs
16:30-17:05	Irene Gil Fernández (University of Warwick)	New lower bounds on kissing numbers and spherical codes in high dimensions

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Oct. 27 Time	**Lecturer**	**Title**
13:35-14:10	Zixiang Xu (Capital Normal University)	On color isomorphic patterns in proper colorings
14:10-14:45	Xichao Shu (Shandong University)	Non-linear Hamilton cycles in linear quasi-random hypergraphs
14:45-15:20	Hehui Wu (Fudan University)	Note on the Turán number of the linear 3-graph C_{13}
Break (10 min)		
15:30-16:05	Felix Joos (Heidelberg University)	Graphs tilings
16:05-16:40	Péter Pél Pach (Budapest University of Technology and Economics)	The Alon--Jaeger--Tarsi conjecture via group ring identities
16:40-17:15	Abhishek Methuku (University of Birmingham)	A proof of the Erdős--Faber--Lovász conjecture

Abstract

Maximum number of almost similar triangles in the plane

Felix Clemen

University of Illinois Urbana-Champaign

A triangle T' is ϵ-similar to another triangle T if their angles pairwise differ by at most ϵ. Given a triangle $T, \epsilon > 0$ and a natural number n, Bárány and Füredi asked to determine the maximum number of triangles being ϵ-similar to T in a planar point set of size n. We determine this quantity for almost all triangles T and sufficiently small ϵ. Exploring connections to hypergraph Turán problems, we use flag algebras and stability techniques for the proof. This is joint work with József Balogh and Bernard Lidický.

New results for MaxCut in H-free graphs

Stefan Glock

ETH Zürich

The MaxCut problem asks for the size $\mathrm{mc}(G)$ of a largest cut in a graph G. It is well known that $\mathrm{mc}(G) \geq m/2$ for any m-edge graph G, and the difference $\mathrm{mc}(G) - m/2$ is called the *surplus* of G. The study of the surplus of H-free graphs was initiated by Erdős and Lovász in the 70s, who in particular asked what happens for triangle-free graphs. This was famously resolved by Alon, who showed that in the triangle-free case the surplus is $\Omega(m^{4/5})$, and found constructions matching this bound.

We prove several new results in this area. First, we obtain an optimal bound when H is an odd cycle, adding to the lacunary list of graphs for which such a result is known. Secondly, we extend the result of Alon in the sense that we prove optimal bounds on

the surplus of general graphs in terms of the number of triangles they contain. Thirdly, we improve the currently best bounds for K_r-free graphs.

Our proofs combine techniques from semidefinite programming, probabilistic reasoning, as well as combinatorial and spectral arguments. Joint work with Oliver Janzer and Benny Sudakov.

Graphs tilings

Felix Joos

Heidelberg University

For every $k \in \mathbb{N}$, Hajnal and Szemerédi determined the optimal minimum degree condition for a graph G that ensures a partition of the vertex set of G into vertex-disjoint copies of K_k; that is, a K_k-tiling of G. This was greatly extended by Kühn and Osthus who resolved this question for any fixed F instead of K_k whenever the order of G is large.

Viewing an F-tiling as a spanning subgraph H where each component is a copy of F, there is the obvious question, what happens if the components are not all isomorphic? We resolve this question completely for bounded degree graphs even if the components are of almost linear order. This includes the resolution of a conjecture of Komlós and shows that the characterization of Kühn and Osthus for arbitrary fixed F holds even for F of order $o(\sqrt{n})$ but is false for some F of order $\omega(\sqrt{n})$.

On color isomorphic patterns in proper colorings

Zixiang Xu

Capital Normal University

For a fixed graph H, what is the smallest number of colors C such that there is a proper edge-coloring of the complete graph K_n with C colors containing no k vertex-disjoint color-isomorphic copies of H? Conlon and Tyomkyn initiated the study of this problem using a variety of combinatorial, probabilistic and algebraic techniques. In this talk, we first review some known results and methods of Conlon and Tyomkyn. Then we will introduce some new results on the even cycle C_4 and the 1-subdivision of complete graph K_t. Finally, several open problems and conjectures will be discussed. Joint work with Gennian Ge, Yifan Jing and Tao Zhang.

Non-linear Hamilton cycles in linear quasi-random hypergraphs

Xichao Shu

Shandong University

In this paper we show that for $\ell < k$ satisfying $(k-\ell) \nmid k$, (p,μ)-denseness plus a minimum $(\ell+1)$-vertex-degree $\alpha n^{k-\ell-1}$ guarantees Hamilton ℓ-cycles, but requiring a minimum ℓ-vertex-degree $\Omega(n^{k-\ell})$ instead is not sufficient. This answers a question of Lenz–Mubayi–Mycroft and characterizes the triples (k,ℓ,d) such that degenerate choices of p and α force ℓ-Hamiltonicity. We actually prove a general result on ℓ-Hamiltonicity in quasi-random k-graphs, assuming a minimum vertex degree and essentially that every two ℓ-sets can be connected by a constant length ℓ-path. This result reduces the ℓ-Hamiltonicity problem to the study of the connection property.

F-factors in quasi-random hypergraphs

Shumin Sun

Shandong University

Given $k \geq 2$ and two k-graphs (k-uniform hypergraphs) F and H, an *F-factor* in H is a set of vertex disjoint copies of F that together covers the vertex set of H. Lenz and Mubayi [*J. Combin. Theory Ser. B, 2016*] studied the F-factor problem in quasi-random k-graphs with minimum degree $\Omega(n^{k-1})$. They posed the problem of characterizing the k-graphs F such that every sufficiently large quasi-random k-graph with constant edge density and minimum degree $\Omega(n^{k-1})$ contains an F-factor, and in particular, they showed that all linear k-graphs satisfy this property.

In this paper we prove a general theorem on F-factors which reduces the F-factor problem of Lenz and Mubayi to a natural sub-problem, that is, the F-cover problem. By using this result, we answer the question of Lenz and Mubayi for those F which are k-partite k-graphs, and for all 3-graphs F, separately. Our characterization result on 3-graphs is motivated by the recent work of Reiher, Rödl and Schacht [*J. Lond. Math. Soc., 2018*] that classifies the 3-graphs with vanishing Turán density in quasi-random k-graphs.

A proof of the Erdős–Faber–Lovász conjecture

Abhishek Methuku

University of Birmingham

The celebrated Erdős–Faber–Lovász conjecture (posed in 1972) states that the chromatic index of any linear hypergraph on n vertices is at most n. In this talk, I will sketch a proof of this conjecture for every large n.

Joint work with D. Kang, T. Kelly, D. Kühn and D. Osthus.

New lower bounds on kissing numbers and spherical codes in high dimensions

Irene Gil Fernández

University of Warwick

Let the *kissing number* $K(d)$ be the maximum number of non-overlapping unit balls in $\mathbb{R}^d$ that can touch a given unit ball. Determining or estimating the number $K(d)$ has a long history, with the value of $K(3)$ being the subject of a famous dispute between Isaac Newton and David Gregory in 1694. We prove that, as the dimension d goes to infinity,

$$K(d) \geq (1+o(1))\sqrt{\frac{3\pi}{8}} \ln \sqrt{\frac{3}{2}} \cdot d^{3/2} \cdot \Big(\frac{2}{\sqrt{3}}\Big)^d,$$

thus improving the previously best known bound of Jenssen, Joos and Perkins by factor $\log(3/2)/\log(9/8)+o(1) = 3.44\ldots$. Our proof is based the novel approach from Joos and Perkins that uses the hard-core sphere model of appropriate fugac ity. Similar constant-factor improvements in lower bounds are also obtained for general spherical codes as well as for the expected density of random sphere packings in the Euclidean space $\mathbb{R}^d$.

Exponential decay of intersection volume with applications on list-decodability and Gilbert–Varshamov type bound

Tuan Tran

The Institute for Basic Science, South Korea

We give some natural sufficient conditions for balls in a metric space to have small intersection. Roughly speaking, this happens when the metric space is (i) expanding and (ii) well-spread, and (iii) certain random variable on the boundary of a ball has a small tail.

As applications, we show that the volume of intersection of balls in Hamming spaces and symmetric groups decays exponentially as their centers drift apart. To verify condition (iii), we prove some deviation inequalities 'on the slice' for functions with Lipschitz conditions.

We then use these estimates on intersection volumes to

- obtain a sharp lower bound on list-decodability of random q-ary codes, confirming a conjecture of Li and Wootters [IEEE Trans. Inf. Theory 2021]; and
- improve Levenshtein's bound from the 70s on constant weight codes by a factor linear in dimension, resolving a problem raised by Jiang and Vardy [IEEE Trans. Inf. Theory 2004].

Our probabilistic point of view also offers a unified framework to obtain improvements on other Gilbert–Varshamov type bounds, giving conceptually simple and calculation-free proofs for q-ary codes, permutation codes, and spherical codes.

This is joint work with Hong Liu and Jaehoon Kim.

Note on the Turán number of the linear 3-graph C_{13}

Hehui Wu

Fudan University

Let the crown C_{13} be the linear 3-graph on 9 vertices $\{a, b, c, d, e, f, g, h, i\}$ with edges $E = \{\{a, b, c\}, \{a, d, e\}, \{b, f, g\}, \{c, h, i\}\}$. Proving a conjecture of Gyárfás et. al., we show that for any crown-free linear 3-graph G on n vertices, its number of edges satisfy $e(G) \leq \frac{3(n-s)}{2}$ where s is the number of vertices in G with degree at least 6. This result, combined with previous work, essentially completes the determination of linear Turán number for linear 3-graphs with at most 4 edges.

This is joint work with Shengtong Zhang in MIT, and Chaoliang Tang, Zeyu Zheng in Fudan University.

The Alon–Jaeger–Tarsi conjecture via group ring identities

Péter Pál Pach

Budapest University of Technology and Economics

The Alon–Jaeger–Tarsi conjecture states that for any finite field $\mathbb{F}$ of size at least 4 and any nonsingular matrix M over $\mathbb{F}$ there exists a vector x such that neither x nor Mx has a 0 component. In joint work with János Nagy we proved this conjecture when the size of the field is sufficiently large, namely, when $61 < |\mathbb{F}| \neq 79$. In this talk we will discuss previous results and sketch our proof.